

$$u \in \bigcap_{a \in A} \text{int } H_{a,0}^+ \quad -1 \quad \sum_{a \in A} \lambda_a = 1 \quad -1 \quad \lambda_a \geq 0 \quad \sum_{a \in A} \lambda_a \cdot a = 0 \quad \text{pk} \quad .k \quad 1$$

$$\Rightarrow \lambda_0 = 0 = \left(\sum_{a \in A} \lambda_a \right) u = \sum_{a \in A} \lambda_a (a \cdot u) > 0 \quad \text{if } a \in A \quad \text{if } u \cdot a > 0 \quad \text{is/c}$$

$$\forall a \in A \quad u \cdot a > \alpha \quad -1 \quad \Rightarrow \quad u \cdot a \text{ p'd'' } \text{ is/c, } 0 \notin \text{conv } A \quad \text{pk} \quad .2$$

$$u \in \bigcap_{a \in A} \text{int } H_{a,0}^+ \quad \text{is/c, } 0 = u \cdot 0 < \alpha \quad !$$

$$\text{is/c} \quad Q = \text{conv} \{b_j\}_{j=1}^m, \quad P = \text{conv} \{a_i\}_{i=1}^n \quad \text{is/c} \quad .k \quad .2$$

$$x = \sum_i \lambda_i a_i \in P \quad \text{pk} \quad \Rightarrow \quad P \times Q = \text{conv} \{ (a_i, b_j) : 1 \leq i \leq n, 1 \leq j \leq m \}$$

$$\text{is/c, } y = \sum_j \mu_j b_j \in Q$$

$$\Rightarrow (x, y) = \sum_{i,j} \lambda_i \mu_j (a_i, b_j)$$

$$\sum_{i,j} \lambda_i \mu_j (a_i, b_j) = \left(\sum_{i,j} \lambda_i \mu_j a_i, \sum_{i,j} \lambda_i \mu_j b_j \right) = \left(\sum_i \lambda_i a_i, \sum_j \mu_j b_j \right) = (x, y)$$

$$Q \text{ is/c } \lambda_k \cdot \alpha \quad G = -1, \quad P \text{ is/c } \lambda_k \cdot \alpha \quad F \quad \text{is/c} \quad \text{is/c} \quad .2$$

$$F = P \cap H_{u,\alpha} \quad P \subset H_{u,\alpha}^+ \quad -1 \quad \Rightarrow \quad (v, \beta), (u, \alpha) \text{ p'd'' } \text{ is/c}$$

$$F \times G = P \times Q \cap H_{(u,v), \alpha+\beta} \quad \text{is/c} \quad G = Q \cap H_{v,\beta}, \quad Q \subset H_{v,\beta}^+ \quad -1$$

$$(x, y) \in H_{(u,v), \alpha+\beta} \cap P \times Q \quad \text{is/c} \quad y \cdot v = \beta, \quad x \cdot u = \alpha \quad \text{is/c} \quad (x, y) \in F \times G \quad \text{pk}$$

$$y \cdot v \leq \beta \quad x \cdot u \leq \alpha \quad \text{is/c} \quad (x, y) \in P \times Q \cap H_{(u,v), \alpha+\beta} \quad \text{pk, } \text{is/c}$$

$$y \cdot v = \beta \quad x \cdot u = \alpha \quad \text{is/c} \quad x u + y v = \alpha + \beta = (x, y) \cdot (u, v) = \alpha + \beta \quad !$$

$$F \times G \ni (x, y) \quad \text{is/c}$$

proof . $f_i(Y) = f_i(X) + 3 f_{i-1}(X)$ izo wll $f_{-1}(Y) = 1$. k . 3

$$F_Y(t) = \sum_{i=0}^{d+1} f_{i-1}(Y) t^{d+1-i} = t^{d+1} + \sum_{i=1}^{d+1} f_{i-1}(Y) t^{d+1-i} =$$

$$= t^{d+1} + \sum_{i=1}^{d+1} [f_{i-1}(X) + 3 f_{i-2}(X)] t^{d+1-i} =$$

$$= t^{d+1} + \sum_{i=1}^{d+1} f_{i-1}(X) t^{d+1-i} + 3 \sum_{i=1}^{d+1} f_{i-2}(X) t^{d+1-i}$$

$$= t \cdot F_X(t) + 3 \sum_{i=0}^d f_{i-1} t^{d-i} = (t+3) F_X(t)$$

$$H_Y(t) = F_Y(t-1) = (t+2) F_X(t-1) = (t+2) H_X(t).$$

F_1, \dots, F_m 'ol $\dim X \geq d-1$ 'o nll . \bar{F}_j nllG nllG Y . 2

$a * F_1, \dots, a * F_m, b * F_1, \dots, b * F_m, c * F_1, \dots, c * F_m$ 'ol . X wll \bar{F}_j nllG . 3

: Y wll \bar{F}_j nllG . 4

$$(*) \left\{ \begin{aligned} \overline{a * F_j} \cap \bigcup_{i < j} \overline{a * F_i} &= a * (\bar{F}_j \cap \bigcup_{i < j} \bar{F}_i) \\ \overline{b * F_j} \cap \left[\bigcup_{i=1}^m \overline{a * F_i} \cup \bigcup_{i < j} \overline{b * F_i} \right] &= \bar{F}_j \cup b * (\bar{F}_j \cap \bigcup_{i < j} \bar{F}_i) \\ \overline{c * F_j} \cap \left[\bigcup_{i=1}^m \overline{a * F_i} \cup \bigcup_{i=1}^m \overline{b * F_i} \cup \bigcup_{i < j} \overline{c * F_i} \right] &= \bar{F}_j \cup c * (\bar{F}_j \cap \bigcup_{i < j} \bar{F}_i) \end{aligned} \right.$$

'o nll X wll \bar{F}_j nllG F_1, \dots, F_m -l nll

. nllG nll (*) - 2 nllG nllG nllG

$$g(z) = (z - (-1))^2 (z - 0)^2 (z - 1)^2 = (z+1)^2 z^2 (z-1)^2 = 0 \cdot z + 1 \cdot z^2 + 0 \cdot z^3 - 2 \cdot z^4 + 0 \cdot z^5 + 1 \cdot z^6$$

• $\forall t \in \mathbb{R} \quad g(t) > 0 \quad \forall t = -1, 0, 1 \quad \text{---} \quad \delta(t) = 0 \quad \text{---} \quad \text{---} \quad \text{---}$

• sk $\mathbb{R}^6 \ni u = (0, 1, 0, -2, 0, 1) \quad \text{---}$

$$\delta(t) \cdot u = g(t)$$

$$\delta(i) \cdot u > 0 \quad \forall i = 0, \pm 1 \quad \text{---} \quad 0 = \delta(i) \cdot u \quad \text{---}$$

• $H_{u,0} \cap P = F \quad \text{---} \quad P \subset H_{u,0} \quad \text{---} \quad \text{---} \quad \text{---}$

$0 \leq i \leq 3 \quad \text{---} \quad h_i = \binom{n-d+i-1}{i} = \binom{2+i}{i} \quad \text{---} \quad \text{---} \quad \text{---} \quad \text{---}$

• $h = (1, 3, 6, 10, 6, 3, 1) \quad \text{---} \quad \text{---} \quad h_i = h_{6-i} \quad \text{---}$

$$f_{k-1} = \sum_{i=0}^k h_i \binom{d-i}{d-k} \quad \text{---}$$

$$f_3 = \sum_{i=0}^4 h_i \binom{6-i}{6-4} = \sum_{i=0}^4 h_i \binom{6-i}{2} \quad \text{---}$$

$$= 1 \cdot \binom{6}{2} + 3 \cdot \binom{5}{2} + 6 \cdot \binom{4}{2} + 10 \cdot \binom{3}{1} + 6 \cdot \binom{2}{0}$$

$$= 117 < \binom{9}{4} = 126$$

$-4 \leq i_1 < \dots < i_4 \leq 4 \quad \text{---} \quad \text{---} \quad \text{---} \quad \text{---}$

• P sk \exists λ sk $\langle \delta(i_1), \dots, \delta(i_4) \rangle = 0 \quad \text{---} \quad \text{---}$