

$$a_n = \left\{ (k_1, k_2, k_3) : k_i \geq 0, \sum_{i=1}^3 k_i = n, 2|k_1, 7|k_2, 0 \leq k_3 \leq 6 \right\}$$

10.1

$$f(x) = \sum_{n=0}^{\infty} a_n x^n = \left( \sum_{i_1=0}^{\infty} x^{2i_1} \right) \left( \sum_{i_2=0}^{\infty} x^{7i_2} \right) \cdot \left( \sum_{i_3=0}^6 x^{i_3} \right) = \frac{1}{1-x^2} \cdot \frac{1}{1-x^7} \cdot \frac{1-x^7}{1-x}$$

$$= \frac{1}{(1-x^2)(1-x)} = \frac{1}{(1-x)^2(1+x)} = \frac{1}{1-x-x^2+x^3}$$

isic  $a_n = \lambda_1 a_{n-1} + \lambda_2 a_{n-2} + \lambda_3 a_{n-3}$  p/c is 11'k) . 2

$$f(x) = \sum_{n=0}^{\infty} a_n x^n = \frac{P(x)}{1-\lambda_1 x - \lambda_2 x^2 - \lambda_3 x^3}$$

isic isf p/c . 2  $\geq \deg P(x)$  11'k)

$$\frac{1}{1-x-x^2+x^3} = \frac{P(x)}{1-\lambda_1 x - \lambda_2 x^2 - \lambda_3 x^3}$$

•  $(\lambda_1, \lambda_2, \lambda_3) = (1, 1, -1)$  -!  $P(x) = 1$  p/c

$n = 7i_2 + i_3$  11'k) 11'k) p/c is 11'k) fo : 1 11'k) . 8

$n = 2i_1 + 7i_2 + i_3$  11'k) 11'k) p/c . 0  $\leq i_2$  , 0  $\leq i_3 \leq 6$  11'k)

• p/c 11'k) p/c 11'k) p/c 11'k) 0  $\leq i_3 \leq 6$  11'k)

•  $a_n = \left\lfloor \frac{n}{2} \right\rfloor + 1$  11'k) , n-1

1)  $n \geq 3$   $\forall x$   $a_n = a_{n-1} + a_{n-2} - a_{n-3}$ ,  $\lambda$  fro  $\lambda^3 = \lambda^2 + \lambda - 1$

$y^3 - \lambda_1 y^2 - \lambda_2 y - \lambda_3 = y^3 - y^2 - y + 1$   $\lambda$  fro  $\lambda^3 = \lambda^2 + \lambda - 1$

$= (y^2 - 1)(y + 1)$

1)  $\theta_1 = -1$ ,  $\theta_2 = 1$ ,  $\theta_3 = 1$

$a_n = (A + B) \cdot \theta_1^n + C \cdot \theta_2^n = A + B + C \cdot (-1)^n$

$$\begin{cases} 1 = a_0 = B + C \\ 1 = a_1 = A + B - C \\ 2 = a_2 = 2A + B + C \end{cases}$$

1)  $(A, B, C) = \frac{1}{4} (2, 3, 1)$

$a_n = \frac{1}{4} ((2n+3) + (-1)^n) = \left\{ \begin{array}{l} \frac{n+1}{2} \quad 2 \nmid n \\ \frac{n+2}{2} \quad 2 \mid n \end{array} \right\} = \lfloor \frac{n}{2} \rfloor + 1$

$\sum_{n=0}^{\infty} a_n x^n = \frac{1}{(1-x)^2(1+x)} = \frac{1}{(1-x)^2} \cdot \frac{1}{1+x}$

$= \sum_{i=0}^{\infty} (i+1) x^i \cdot \sum_{j=0}^{\infty} (-1)^j x^j = \sum_{n=0}^{\infty} \left( \sum_{i+j=n} (i+1) \cdot (-1)^j \right) x^n$

$a_n = \sum_{i=0}^n (i+1) (-1)^{n-i} = (-1)^n (1 - 2 + 3 - 4 + \dots + (-1)^n (n+1))$   
 $= \lfloor \frac{n}{2} \rfloor + 1$

$$g(n, k) = \left| \{ (i_1, \dots, i_k) : 1 \leq i_1 < i_2 - 1 < i_3 - 2 < \dots < i_k - (k-1) \leq n-1 - (k-1) = n-k \} \right|$$

$$= \left| \{ (j_1, \dots, j_k) : 1 \leq j_1 < j_2 < \dots < j_k \leq n-k \} \right| = \binom{n-k}{k}$$

100  $1 \leq i \leq n-1$  - f . a

$$A_i = \{ (x_1, \dots, x_n) \in \{1, 2, 3\}^n : (x_i, x_{i+1}) = (1, 2) \}$$

15k ,  $1 \leq i_1 < \dots < i_k \leq n-1$  101

$$\left| \bigcap_{j=1}^k A_{i_j} \right| = \begin{cases} 3^{n-2k} & i_1 < i_2 - 1 < \dots < i_k - (k-1), \\ 0 & \text{otherwise.} \end{cases}$$

$$\left| \bigcup_{i=1}^{n-1} A_i \right| = \sum_{k=1}^{n-1} (-1)^{k+1} g(n, k) \cdot 3^{n-2k} = \sum_{k=1}^{n-1} (-1)^{k+1} \binom{n-k}{k} \cdot 3^{n-2k}$$

101k0  $\{1, 2, 3\}$  p101000 n n p1002 n1000 n000 101

$$3^n - \left| \bigcup_{i=1}^{n-1} A_i \right| = \sum_{k=0}^{\lfloor \frac{n}{2} \rfloor} (-1)^k \binom{n-k}{k} 3^{n-2k}$$

$$\begin{array}{l}
 b_{n-1} \quad (c) \quad 1 - \lambda \quad \text{מילוי } (b_{n-1}) \\
 b_{n-1} \quad \text{"} \quad 3 - \lambda \quad \text{"} \quad \text{"} \quad \text{"} \quad \text{"} \quad \text{"} \\
 (c) \quad 2 - \lambda \quad \text{"} \quad \text{"} \quad \text{"} \quad \text{"} \quad \text{"}
 \end{array}$$

.2  
(הנחה)

$$b_{n-1} - \left( \begin{array}{c} \text{מילוי } (b_{n-1}) \\ 1 \quad \lambda \quad \text{מילוי } (b_{n-1}) \end{array} \right) = b_{n-1} - b_{n-2}$$

$$b_n = 3b_{n-1} - b_{n-2} \quad | \text{כפ}$$

p)  $y^2 - 3y + 1 = 0$       נ"ל פירוק

$$\theta_1 = \frac{3 + \sqrt{5}}{2} \quad \theta_2 = \frac{3 - \sqrt{5}}{2}$$

$$b_n = A \theta_1^n + B \theta_2^n \quad | \text{כפ}$$

$$\Leftrightarrow \begin{cases} 1 = b_0 = A + B \\ 3 = b_1 = A \theta_1 + B \theta_2 \end{cases} \quad | \text{כפ}$$

$$(A, B) = \left( \frac{5 + 3\sqrt{5}}{10}, \frac{5 - 3\sqrt{5}}{10} \right)$$

|כפ

$$b_n = \left( \frac{5 + 3\sqrt{5}}{10} \right) \left( \frac{3 + \sqrt{5}}{2} \right)^n + \left( \frac{5 - 3\sqrt{5}}{10} \right) \left( \frac{3 - \sqrt{5}}{2} \right)^n$$

$$16 = 2(9-1) = \sum_{i=1}^9 \deg_T(i) = 4+3+3 + \sum_{i=4}^9 \deg_T(i) = 10 + \sum_{i=4}^9 \deg_T(i)$$

$4 \leq i \leq 9$  or  $\deg_T(i) = 1$  or  $\sum_{i=4}^9 \deg_T(i) = 6$   $\Leftarrow$   
 (1)  $\{3\}$   $\{3,3\}$  or  $\{6\}$

$$\binom{n-2}{d_1, \dots, d_{n-1}} = \binom{7}{3; 2; 2; 0; \dots; 0} = \frac{7!}{3! 2! 2!} = 210$$

like  $\{1,2,3\}$  [n]  $\{1,2,3\}$  and  $\{1,2,3\}$   $\rightarrow$  mod 7

like  $\{i,j,k\}$  and  $\{i,j,k\}$ ,  $|A_{ij}| = 2 \cdot n^{n-3}$  like  $\{i,j\}$  and  $\{i,j\}$

$$|A_{ij} \cap A_{jk}| = 3 \cdot n^{n-4}, \quad A_{ij} \cap A_{jk} \cap A_{ik} = \emptyset$$

$$\left| \bigcup_{1 \leq i < j < k \leq 3} A_{ij} \right| = 3 \cdot (2 \cdot n^{n-3}) - 3 \cdot (3 \cdot n^{n-4}) \quad |> 6$$

like  $\{1,2,3\}$  and like  $\{1,2,3\}$  and  $\{1,2,3\}$  or  $\{1,2,3\}$

(1)  $\{1,2\}, \{1,3\}, \{2,3\}$

$$n^{n-2} - 6 \cdot n^{n-3} + 9 \cdot n^{n-4}$$

1)  $C_j = A_{\lfloor \frac{j-1}{3} \rfloor + 1}$   $C_1, \dots, C_{3n}$   $n \cdot 3 \cdot 3 = 9$  . 4

$$C_1 = C_2 = C_3 = A_1, C_4 = C_5 = C_6 = A_2, \dots, C_{3n-2} = C_{3n-1} = C_{3n} = A_n$$

$$I \subset [3n]$$

$$I = \{i \in \mathbb{N} : \{3i-2, 3i-1, 3i\} \cap J \neq \emptyset\} = \left\{ \left\lfloor \frac{j-1}{3} \right\rfloor + 1 : j \in J \right\}$$

$$\bigcup_{j \in J} C_j = \bigcup_{i \in I} A_i \quad -1 \quad |I| \geq \frac{|J|}{3} \quad \text{1st}$$

$$|\bigcup_{j \in J} C_j| = |\bigcup_{i \in I} A_i| \geq 3|I| \geq |J| \quad \text{pf}$$

- e p p'le  $\{x_j\}_{j=1}^{3n}$   $n \cdot 3$  Hall  $\{0, 1\}$   $\text{pf}$

$$B_i = \{x_{3i-2}, x_{3i-1}, x_{3i}\} \quad 1 \leq i \leq n \quad \text{1st} \quad 1 \leq j \leq 3n \quad \text{of } x_j \in C_j$$

$$1 \leq i \neq i' \leq n \quad \text{of } B_i \cap B_{i'} = \emptyset \quad -1 \quad B_i \subset A_i \quad n \cdot 3 \cdot 3$$

לכאן  $N = 10 \binom{10^2}{R(10, \dots, 10) - 1} + 1$  (כא) . 2.5

הפונקציה  $f: [N] \times [N] \rightarrow [10]$

$|M| \geq R \binom{10^2}{10, \dots, 10}$  ,  $M \subset [N]$  נ"ק ר"ב קבועים

$f(m, m) = a$   $\forall m \in M$  כן,  $a \in [10]$  !

$M$  קבוצת קבוצות  $G$  פשוטה  $f$  היא פונקציה  $f$  נגזרת

$m < m'$   $\phi(\{m, m'\}) = \{f(m, m'), f(m', m)\}$  "ר

$|M| \geq R \binom{10^2}{10, \dots, 10}$  !  $n$  קבוצות  $10^2$  "ר קבוצות  $15$

$\exists$   $b, c \in [10]$  !  $|A| = 10$   $A \subset M$  נ"ק ר"ב "ר

$m \neq m' \in M$   $\forall$   $\phi(\{m, m'\}) = \{b, c\}$

$\forall m, m' \in M$   $\forall$   $b, c \in [10]$   $\phi$  נ"ק ר"ב "ר

$$f(m, m') = \begin{cases} a & m = m' \\ b & m < m' \\ c & m > m' \end{cases}$$