

প্রদর্শন করা হবে  $[3]^n$  -> একটি সময় এখন  $b_n = ?$  | Noj . k. 1

$$b_n = 2a_{n-1}$$

এসে . ৩ -> ১ক ২ -> নিম্নলিখিত

$$\boxed{a_n = 2a_{n-1} + 2a_{n-2}}$$

$$, \quad a_n = b_n + b_{n-1} = 2(a_{n-1} + a_{n-2})$$

$$\Rightarrow (p.p) \quad a_1 = 3, \quad a_0 = 1 \quad -1 \quad \text{পর্যন্ত} \quad .$$

$$f(x) = \sum_{n=0}^{\infty} a_n x^n = 1 + 3x + \sum_{n=2}^{\infty} 2(a_{n-1} + a_{n-2}) x^n =$$

$$= 1 + 3x + 2x \sum_{n=2}^{\infty} a_{n-1} x^{n-1} + 2x^2 \sum_{n=2}^{\infty} a_{n-2} x^{n-2} =$$

$$= 1 + 3x + 2x(f(x) - 1) + 2x^2 f(x) = 1 + x + (2x + 2x^2) f(x)$$

$$\boxed{f(x) = \frac{1+x}{1-2x-2x^2}}$$

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$$y^2 - 2y - 2 = 0$$

ক'র ব্যর্থ দ্রোজ এন্ডিজ নিম্নলিখিত করিবে .

$$\frac{2 \pm \sqrt{4+8}}{2} = 1 \pm \sqrt{3} \quad \text{প্রাপ্ত এন্ডিজুড়ে}$$

$$N.N.N.A.B \quad \text{সুর} \quad a_n = A(1+\sqrt{3})^n + B(1-\sqrt{3})^n$$

পি

$$1 = a_0 = A + B$$

$$3 = a_1 = A(1+\sqrt{3}) + B(1-\sqrt{3})$$

$$(p.1) \quad (A, B) = \left( \frac{\sqrt{3}+2}{2\sqrt{3}}, \frac{\sqrt{3}-2}{2\sqrt{3}} \right) \quad \text{পি}$$

$$\boxed{a_n = \frac{\sqrt{3}+2}{2\sqrt{3}} (1+\sqrt{3})^n + \frac{\sqrt{3}-2}{2\sqrt{3}} (1-\sqrt{3})^n}$$

15k,  $V_{n-1/k}$  נולב בז'ר פט  $[n]$  כר' ה' F פ' 2

$|V_t| \cdot |V_t| \cdot n^{t-2}$  (כ' ה' F נול פ' ג' ג'  $[n]$  כר' פ' ג' ג' כ' (x)

M נול פ' ג' ג'  $[n]$  כ' ג' ג' כ' (x) כ' .lc

$$2^{\frac{n}{2}} \cdot n^{\frac{n}{2}-2} \quad \text{ל'}$$

[n] נ' ג' ג' נ' נ' נ'  $A_i$  ס' נ' ,  $e_i = \{2i-1, 2i\}$  כ' .p

; (x) כ' ג' . e; נול פ' ג' ג'

$$\left| \bigcap_{i \in I} A_i \right| = 2^{|I|} \cdot n^{[|I| + (n - 2|I|)] - 2} = 2^{|I|} \cdot n^{n-|I|-2}$$

$$\left| \bigcup_{i=1}^n A_i \right| = \sum_{k=1}^n (-1)^{k+1} \binom{n}{k} \cdot 2^k \cdot n^{n-k-2}$$

(c)  $E \cap M = \emptyset$  נ' ג' ג' T =  $([n], E)$  {ג' ג'} כ' פ'

$$n^{n-2} - \left| \bigcup_{i=1}^n A_i \right| = \sum_{k=0}^n (-1)^k \binom{n}{k} 2^k \cdot n^{n-k-2}$$

$$= n^{n-2} \sum_{k=0}^n \binom{n}{k} \left(\frac{-2}{n}\right)^k = n^{n-2} \left(1 - \frac{2}{n}\right)^n$$

$$= n^{\frac{n}{2}-2} (n-2)^{\frac{n}{2}}$$

$$l = \max_{b \in B} \deg_G(b), \quad k = \min_{a \in A} \deg_G(a) \quad (\text{NO}) \quad .A.3$$

: p" jn Hall 1 [k] & p[32] . l \leq k \quad \text{וגם לא}

לול, I \subset A \quad (\text{KA})

$$\begin{aligned} k \cdot |I| &\leq \sum_{a \in I} \deg_G(a) = |\{(a, e) : a \in I, a \in e \in E\}| \leq \\ &\leq |\{(b, e) : b \in \Gamma(I), b \in e \in E\}| \\ &= \sum_{b \in \Gamma(I)} \deg_G(b) \leq l \cdot |\Gamma(I)| \end{aligned}$$

$$\therefore |\Gamma(I)| \geq \frac{k}{l} |I| \times |I| \quad (\text{פ''ל})$$

. A \{3\} \cap \{1, 2, 4\} \text{ כ' } \{1, 2, 4\} \text{ כ' } G \text{ פ'}

$$\text{לט}, \mathbb{Z}_{10} \rightarrow \text{עליה} \left\{ \left( \sum_{i=1}^c x_i \pmod{10} \right)_{c=0}^{100} \right\} \text{ כ' } \mathbb{Z}_{10}^{100} \quad .4$$

$$|C| = \left[ \frac{101}{10} \right] = 11 \quad \text{לט} \quad C \subset \{0, \dots, 100\} \quad \text{נ' } p \text{ מ' } p$$

$$\left( \sum_{i=1}^c x_i \pmod{10} \right) = k \quad - \text{ט' } p \quad k \in \mathbb{Z}_{10} \quad -1$$

$$15/6 \quad C = \{c_1, c_{11}\} \quad (\text{NO}) \quad c \in G \quad (\text{פ'ל})$$

$$\left( \sum_{i=1}^{c_1} x_i \pmod{10} \right) = \dots = \left( \sum_{i=1}^{c_{11}} x_i \pmod{10} \right) = k.$$

נ"מ  $1 \leq i \leq 10$   $\sum f_i \leq 10$   $\sum f_i = 10$

$$\sum_{j=c_i+1}^{c_{i+1}} x_j \geq 0 \quad \text{ול } \sum_{j=c_i+1}^{c_{i+1}} x_j \equiv 0 \pmod{10}$$

$$1 \leq i \leq k-1 \quad \sum_{j=c_i+1}^{c_{i+1}} x_j \geq 20 \quad \text{ול } \sum_{j=c_i+1}^{c_{i+1}} x_j \neq 10 \quad \text{ול } f_i \neq 0$$

$$\sum_{j=c_0+1}^{c_1} x_j = 10 \cdot 20 \leq \sum_{j=c_0+1}^{c_1} x_j \leq \sum_{j=1}^{100} x_j = 199$$

$$\sum_{j=c_i+1}^{c_{i+1}} x_j = 10 \quad \text{ול } 1 \leq i \leq k-1 \quad \text{ול } f_i \neq 0$$

$$(y_1, \dots, y_{100}) = (\underbrace{1, 1, 1, 1, 1, 1, 1, 1, 1, 1}_{10}, \underbrace{1, 1, 1, 1, 1, 1, 1, 1, 1, 1}_{10}, \dots, \underbrace{1, 1, 1, 1, 1, 1, 1, 1, 1, 1}_{10})$$

הנ"מ  $\sum f_i \geq 10$   $\forall f \in F$   $\sum f_i = 10$   $\forall f \in F$   $f_i \neq 0$   $\forall f \in F$   $f_i > 0$

$|f| \geq f_i \geq 1 \forall f \in F$   $\forall f \in F \quad f_i > 0$   $\forall f \in F \quad f_i \neq 0$

$\sum f_i \geq 10 \quad \text{ול } f_i > 0 \quad \text{ול } f_i \neq 0$

$$\text{הנ"מ } 2|E| = |\{(e, f) \in E \times F : e \in f\}| = \sum_{f \in F} |f| \geq 6 \cdot |F|$$

$$2|E| = |V| - |E| + |F| \leq |V| - |E| + \frac{|E|}{3} \leq |V| - \frac{2|E|}{3}$$

$$3|V| \leq \sum_{v \in V} \deg v = 2|E| \leq 3(|V| - 2)$$

הנ"מ

$$(\text{****}) \quad 2|E| = \sum_{v \in V} \deg(v) = 3|V|$$

Pf

$$1 = |V| - |E| + |F| = \frac{2}{3}|E| - |E| + |F| = |F| - \frac{1}{3}|E|$$

$$(*). \quad 12 = 6|F| - \frac{1}{3}|E|$$

Pf

Sei  $\{V(F_i)\}$  ist der kleinste Schnitt, der  $F_i$  trennt  $\rightarrow$  10)

$$(**) \quad 2|E| = 6|F| - 12 = 6|F_6| + 6|F_4| - 12$$

$$(\text{***}) \quad 2|E| = \sum_{i=3}^6 |F_{ik}| - 4|F_4| + 6|F_6|$$

$\Rightarrow$  11)  $(\text{***}) \rightarrow (\text{**})$  und

$$\boxed{|F_4|=6} \quad \Leftarrow \quad 2|F_4| - 12 = 0$$

$$\boxed{|F_6|=8} \quad \Leftarrow \quad \boxed{|F|=14} \quad \Leftarrow \quad \boxed{|E|=36} \quad \Leftarrow \quad |V|=24$$