

$f(x,y) = e^x(\cos y, \sin y)$ "if ∇f $f: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ then .10.1

plc , f ∇f $\nabla f(0,0)$ sic

$$\det Df(x,y) = e^x \neq 0 \quad \text{pfi} \quad Df(x,y) = \begin{bmatrix} e^x \cos y & -e^x \sin y \\ e^x \sin y & e^x \cos y \end{bmatrix}$$

, $\mathbb{R}^2 \ni (x,y)$ $\nabla f(x,y)$ $Df(x,y)$ pfi

"if $g: \mathbb{R}^n \rightarrow \mathbb{R}$ $\forall i \in \{1, \dots, n\}$ $(b_{i+1}, b_n) = b \in \mathbb{R}^n$ then .11

$$\lim_{|x| \rightarrow \infty} |f_i(x)| = \infty \quad g(x) = |f(x) - b|^2 = \sum_{i=1}^n (f_i(x) - b_i)^2$$

(c) $\exists r, \forall |x| \geq r \quad \exists \delta \quad |f(x)| > 2|b| - \delta \quad \forall r > 0 \quad \exists \delta \quad \forall |x| \geq r \quad -\delta < g(x) < \delta$

- δ $\forall a \in K$ $|a| < r \quad K = \{x: |x| \leq r\}$

$$|f(a) - b|^2 = g(a) = \min \{g(x): x \in K\}$$

sic $a: |x|=r$ plc $\Rightarrow K \cap \{x: |x|=r\}$

$$g(x) = |f(x) - b|^2 \geq (|f(x)| - |b|)^2 > |b|^2 = g(0)$$

$$\left(\frac{\partial g}{\partial x_1}(a), \dots, \frac{\partial g}{\partial x_n}(a) \right) = Dg(a) = 0 \quad \text{pfi at int K}$$

$$1 \leq j \leq n \quad \text{pfi} \quad \frac{\partial g}{\partial x_j}(a) = \sum_{i=1}^n 2 \cdot (f_i(a) - b_i) \frac{\partial f_i}{\partial x_j}(a)$$

$$\nabla f(a) \cdot (f(a) - b) = Df(a) = 0 \quad \text{pfi}$$

$$f(a) = b \quad \text{pfi}$$

$$g(x_1, y_1, z) = (x_1 + y_1 + z, x_1^2 + y_1^2 + z^2 - 1), \quad f(x_1, y_1, z) = x$$

$$C = g^{-1}(0)$$

'sk

$$x=y=z \text{ minc } u \in C \text{ so } \operatorname{rk} Dg(u)=2 \quad \text{pf}, \quad Dg(x_1, y_1, z) = \begin{bmatrix} 1 & 1 & 1 \\ 2x & 2y & 2z \end{bmatrix}$$

pk b) x_1, y_1, z surj fib $\{0\}$ minc $C \ni (x_1, y_1, z)$

$$'sk \quad f(u) = \max \{f(v) : v \in C\} \quad \text{min} \quad C \ni u = (x_0, y_0, z_0)$$

$$(1, 0, 0) = Df(u) = [x_1 \ y_1] \quad Dg(u) = (\lambda_1 + 2\lambda_2 x_0, \lambda_1 + 2\lambda_2 y_0, \lambda_1 + 2\lambda_2 z_0)$$

$$0 = \lambda_1 + 2\lambda_2 y_0 = \lambda_1 + 2\lambda_2 z_0 \quad -! \quad L = \lambda_1 + 2\lambda_2 x_0$$

$$\text{minc}, \lambda_1 = 1 \text{ pt! } \lambda_1 = 0 \text{ 'sk} \quad \lambda_2 = 0 \quad \text{islo} \quad y_0 \neq z_0 \quad \text{pk}, \quad \lambda_2 y_0 = \lambda_2 z_0$$

$$\text{pf}, \quad x_0^2 + 2y_0^2 = 1, \quad x_0 + 2y_0 = 0 \quad \text{pf}, \quad y_0 = z_0 \quad \text{pf}$$

$$f(u) = x_0 = \sqrt{\frac{2}{3}} \quad \Leftarrow y_0 = z_0 = \pm \frac{1}{\sqrt{6}} \quad \Leftarrow L = x_0^2 + 2y_0^2 = 4y_0^2 + 2y_0^2 = 6y_0^2$$

$$\text{minc} \quad C \ni (x_1, y_1, z) \quad \text{pf} \quad Dg(x_1, y_1, z) = 2 \quad -! \quad \text{minc}$$

(a) $C \ni (x_1, y_1, z)$ $\Leftrightarrow C - \{ (0, 0, 0) \}$ \Rightarrow

$$'sk \quad T_C \ni (1, 1, -3) \quad \text{pk}, \quad \ker Dg(x_1, y_1, z) = \ker \begin{bmatrix} 1 & 1 & 1 \\ 2x & 2y & 2z \end{bmatrix}$$

$$\text{pf}, \quad x_1 + 2y_1 - 3z_1 = 0 \quad \text{pf}, \quad \begin{bmatrix} 1 & 1 & 1 \\ 2x & 2y & 2z \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\text{pf}, \quad \begin{bmatrix} 1 & 2 & -3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \text{if}, \quad x_1 + y_1 + z_1 = 0 \quad \text{ptl} \quad \text{pf}$$

$$(x_1, y_1, z_1) = \underbrace{(-5, 4, 1)}_{(x_1, y_1, z_1)}$$

$$\text{pf}, \quad (x_1, y_1, z_1) = \lambda(-5, 4, 1)$$

.k .3

$$N_T(r, \theta) = \frac{\partial T}{\partial r} \times \frac{\partial T}{\partial \theta} = \begin{pmatrix} (\cos \theta, \sin \theta, \frac{\partial f}{\partial r}) \\ (-r \sin \theta, r \cos \theta, \frac{\partial f}{\partial \theta}) \end{pmatrix}$$

$$= \left(\sin \theta \frac{\partial f}{\partial \theta} - r \cos \theta \frac{\partial f}{\partial r}, -r \sin \theta \frac{\partial f}{\partial r} - r \cos \theta \frac{\partial f}{\partial \theta}, r \right)$$

$$|N_T(r, \theta)| = \left(\left(\frac{\partial f}{\partial \theta} \right)^2 + r^2 \left(\frac{\partial f}{\partial r} \right)^2 + r^2 \right)^{\frac{1}{2}}$$

$$\boxed{\text{area}(S) = \int_{r=0}^L \int_{\theta=0}^{2\pi} \left(r^2 + r^2 \left(\frac{\partial f}{\partial r} \right)^2 + \left(\frac{\partial f}{\partial \theta} \right)^2 \right)^{\frac{1}{2}} d\theta dr}$$

$$\text{area}(S) = \int_{r=0}^L \int_{\theta=0}^{2\pi} \left[r^2 + r^2 \left(\frac{\partial f}{\partial r} \right)^2 + \left(\frac{\partial f}{\partial \theta} \right)^2 \right]^{\frac{1}{2}} d\theta dr$$

$$= \int_{r=0}^L \int_{\theta=0}^{2\pi} (r^2 + \theta^2 + 2r\theta)^{\frac{1}{2}} dr d\theta = \int_{r=0}^L \int_{\theta=0}^{2\pi} (r+\theta) dr d\theta$$

$$= \int_{\theta=0}^{2\pi} \left(\int_{r=0}^L r dr \right) d\theta + \int_{r=0}^L \left(\int_{\theta=0}^{2\pi} \theta d\theta \right) dr = \boxed{\pi + 2\pi^2}$$

$$(P_1, Q) = f = \frac{(ax-y, bx+4y)}{x^2+4y^2}$$

1st F.S. , $\frac{\partial Q}{\partial x} = \frac{\partial P}{\partial y} \Leftrightarrow$ 1st F.S. F .k

$$\frac{-bx^2+4by^2-8xy}{x^2+4y^2} = \frac{b(x^2+4y^2)-(bx+4y)2x}{x^2+4y^2} = \frac{\frac{\partial Q}{\partial x} = \frac{\partial P}{\partial y}}{x^2+4y^2} = -\frac{(x^2+4y^2)-(ax-y)8y}{x^2+4y^2} =$$

$$= \frac{-x^2+4y^2-8axy}{x^2+4y^2}$$

$a=b=1$ $\int_{\Gamma} \mathbf{F} \cdot d\mathbf{r}$ \rightarrow $\int_{\Gamma} \mathbf{F} \cdot \mathbf{n} ds$

$$\mathbf{F} = \left(\frac{x-y}{x^2+y^2}, \frac{x+4y}{x^2+y^2} \right)$$

|p|

Given Γ , find $\int_{\Gamma} \mathbf{F} \cdot d\mathbf{r}$. A plan is to use \mathbf{F} to find $\mathbf{F} \cdot \mathbf{n}$.

$$\Gamma(t) = (2\cos t, \sin t) \quad \text{for } t \in [0, \pi] \quad \text{and } \Gamma: [0, \pi] \rightarrow A$$

$$\int_{\Gamma} \mathbf{F} \cdot d\mathbf{r} = \int_{t=0}^{2\pi} \frac{(2\cos t - \sin t, 2\cos t + 4\sin t)}{(2\cos t)^2 + (\sin t)^2} \cdot (-2\sin t, \cos t) dt$$

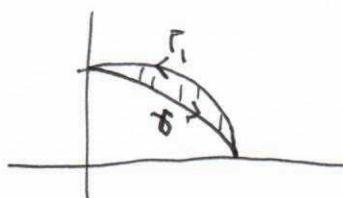
|s/c

$$= \frac{1}{4} \int_{t=0}^{2\pi} (2\sin^2 t + 2\cos^2 t) dt = \frac{4\pi}{4} = \pi \neq 0$$

$$0 \leq t \leq \pi/2$$

$$\Gamma_1(t) = (2\cos t, \sin t)$$

|n.v| .



$\text{arc length } |\Gamma_1|$ \times $\|\mathbf{F}\|$

$-\Gamma_1$ \Rightarrow \mathbf{F}

|p|

$$\int_{\Gamma} \mathbf{F} \cdot d\mathbf{r} = - \int_{\Gamma_1} \mathbf{F} \cdot d\mathbf{r} \stackrel{\leftarrow}{=} - \int_{t=0}^{\pi/2} \mathbf{F}(\Gamma_1(t)) \cdot \dot{\Gamma}_1(t) dt$$

$$\begin{aligned} & \text{fix } \mathbf{F} \text{ at } \mathbf{n} \\ & = - \int_{t=0}^{\pi/2} \frac{1}{2} dt = \boxed{-\frac{\pi}{4}} \end{aligned}$$

$$B = \{(r \cos \theta, r \sin \theta, 0) : 0 \leq r \leq a, 0 \leq \theta \leq 2\pi\} \quad b.5$$

$$T: [0, a] \times [0, 2\pi] \times [0, \pi] \rightarrow K \quad \text{bijection and } p$$

" " \rightarrow

$$T(r, \theta, \phi) = (r \cos \theta, (1+r \sin \theta) \cos \phi, (1+r \sin \theta) \sin \phi)$$

K is a ball in \mathbb{R}^3

$$DT(r, \theta, \phi) = \begin{bmatrix} \cos \theta & \sin \theta \cos \phi & \sin \theta \sin \phi \\ -r \sin \theta & r \cos \theta \cos \phi & r \cos \theta \sin \phi \\ 0 & -(1+r \sin \theta) \sin \phi & (1+r \sin \theta) \cos \phi \end{bmatrix}$$

$$J_T(r, \theta, \phi) = \det DT(r, \theta, \phi) = r(1+r \sin \theta) \quad p$$

|p|

$$\text{vol}(K) = \iiint_{r=0}^a \int_{\theta=0}^{2\pi} \int_{\phi=0}^{2\pi} |J_T(r, \theta, \phi)| dr d\theta d\phi = \int_{r=0}^a \int_{\theta=0}^{2\pi} \int_{\phi=0}^{2\pi} r(1+r \sin \theta) d\phi d\theta dr$$

$$= \int_{\theta=0}^{2\pi} \int_{\phi=0}^{2\pi} \left[\int_{r=0}^a r dr \right] d\phi d\theta = (2\pi)^2 \cdot \frac{a^2}{2} = \boxed{2\pi^2 a^2}$$

$$\int_K dx dy dz = \text{vol}(K) \stackrel{\text{def}}{=} \int_K d\sigma = \int_K d\sigma F dx dy dz$$

$$= \int_K ((13 + d^2 + z^2) + (2z^2 - 10d) + (13 - 3z^2)) dx dy dz$$

$$= \int_K (26 - 10d + d^2) dx dy dz = (1 + (d-5)^2) \text{vol}(K)$$

$$\boxed{d=5}$$

\Leftarrow